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Implications of anomalous U(1) symmetry in unified models: the flipped $SU(5) \times U(1)$ paradigm

John Ellis

Theory Division, CERN, CH 1211 Geneva 23, Switzerland

George K. Leontaris and John Rizos

Theoretical Physics Division, Ioannina University, GR-45110 Ioannina, Greece
E-mail: leonta@cc.uoi.gr, irizos@cc.uoi.gr

Abstract: A generic feature of string-derived models is the appearance of an anomalous abelian $U(1)_A$ symmetry which, among other properties, constrains the Yukawa couplings and distinguishes the three families from each other. In this paper, we discuss in a model-independent way the general constraints imposed by such a $U(1)_A$ symmetry on fermion masses, R-violating couplings and proton-decay operators in a generic flipped $SU(5) \times U(1)'$ model. We construct all possible viable fermion mass textures and give various examples of effective low-energy models which are distinguished from each other by their different predictions for B-, L- and R-violating effects. We pay particular attention to predictions for neutrino masses, in the light of the recent Super-Kamiokande data.

Keywords: Discrete and Finite Symmetries, Anomalies in Field and String Theories, Supersymmetric Effective Theories, GUT.

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Contents

1. Introduction

The minimal supersymmetric extension of the Standard Model (MSSM) is a popular extension of the Standard Model (SM) of the electroweak and strong interactions. The MSSM is the simplest extension of the SM which solves the naturalness aspect of the hierarchy problem. Moreover, it predicts cold dark matter with a density that could well be in the range favoured by astrophysical observations [1], and a relatively light Higgs boson, as hinted by the precision electroweak data from LEP and elsewhere [2]. Moreover, several supersymmetric grand unified (SUSY GUT) or partially unified extensions of the MSSM, based on gauge groups like SU(4), SU(5), SO(10), etc. [3], provide successful predictions of the electroweak mixing angle. This is based on an appealing unification of the strong and electroweak forces at an energy scale of the order of 10^{16} GeV. Such SUSY GUT extensions of the Standard Model also suggest the possibility of unifying the above three forces with gravity in the context of a supergravity or superstring scenario close to the Planck scale. Since this picture provides a rather promising framework for a more complete theory, we think it appropriate to take a step further by analyzing in more detail various additional phenomenological questions.

Among the large number of questions to be answered in a more complete model of elementary-particle interactions is the origin and the strength of the Yukawa couplings. As is well known, the fermion masses arise in the MSSM from the Yukawa couplings in the superpotential through the vevs of the two Higgs doublets h_d, h_u which couple to quarks and leptons to form mass terms as follows:

$$\mathcal{W} = \lambda^u_{ij}\, q_i\, u^c_j\, h_u + \lambda^d_{ij}\, q_i\, d^c_j\, h_d + \lambda^e_{ij}\, \ell_i\, e^c_j\, h_d\,. \tag{1.1}$$

There are several basic questions about the nature of the above couplings which one may address:

(1) Although all these terms are invariant under the (MS)SM gauge group, there is no explanation why some of the Yukawa couplings are much smaller than others, as is required in order to account for the fermion mass hierarchy.

(2) In addition to the standard Yukawa couplings which provide masses for the quarks and leptons, the gauge symmetry and supersymmetry of the MSSM also allow terms which violate baryon and lepton number already at the renormalizable level, namely:

$$\mathcal{W} = \lambda_{ijk}\, \ell_i\, \ell_j e^c_k + \lambda'_{ijk}\, \ell_i\, q_j\, d^c_k + \lambda''_{ijk}\, u^c_i\, d^c_j\, d^c_k + \mu_i\, \ell_i\, h_u\,. \tag{1.2}$$

It would therefore be natural to allow also their existence in the Yukawa lagrangian. In order, however, to avoid rapid baryon decay induced by these interactions, any combination of the λ' and λ'' couplings would have to be extremely small.

(3) If the MSSM is to be embedded in a more complete theory which includes gravity, non-renormalizable terms will be induced, suppressed by powers of the Planck mass or some related scale. Some of these terms would have important observational effects, such as the $QQQ\ell$ operator, which cannot be eliminated by postulating conservation of R parity [4].

The most appropriate strategy for allowing some of the Yukawa couplings in the lagrangian while forbidding others is to postulate additional symmetries. In particular, with regard to the problem (2) mentioned above, the unwanted baryon- and lepton-number-violating interactions given by the first three terms of (1.2) would be prevented by the matter parity, known as R parity [5], which is defined as

$$R = (-1)^{(3B+L+2S)}\,, \tag{1.3}$$

where B, L and S are the baryon, lepton and spin quantum numbers. Under this symmetry, all SM particles are even, whilst their superpartners are odd. Imposing this R-parity symmetry, all dangerous terms in (1.2) change sign and are eliminated

Constrained R Couplings	Reaction
$\lambda_{1k1}\lambda_{1k2},\lambda_{231}\lambda_{131} < 7\times 10^{-7}$	$\mu \to 3e$
$\lambda'_{1k1}\lambda'_{2k1},\, \lambda'_{11k}\lambda'_{21k} < 5\times 10^{-8}$	$\mu\, Ti \to e\, Ti$
$\lambda'_{1k1}\lambda'_{2k2} < 8\times 10^{-7}$	$K_L \to \mu e$
$\lambda'_{i12}\lambda'_{i21} < 10^{-9}$	Δm_K
$\lambda'_{k13}\lambda'_{k31} < 8\times 10^{-8}$	Δm_B
$\mathrm{Im}[\lambda'_{k12}\lambda'^{*}_{k21}] < 8\times 10^{-12}$	ϵ_K
$\lambda'_{11k}\lambda''_{11k} < 10^{-24}, k = 2,3$	$p \to \pi^+(K^+)\bar\nu$
$\lambda'_{ijk}\lambda''_{\ell mn} < 10^{-9}$	$p \to \pi^+(K^+)\bar\nu$
$\lambda''_{112}\lambda_{\ell 33,3m3} < 10^{-21}$	$p \to K^+\bar\nu$
$\lambda''_{112}\lambda_{\ell 22,2m2} < 10^{-20}$	$p \to K^+\bar\nu$
$\lambda''_{112}\lambda_{\ell 11,1m1} < 10^{-17}$	$p \to K^+\bar\nu$
$\lambda''_{112}\lambda_{123,213} < 10^{-14}$	$p \to K^+ + 3\nu$
$\lambda''_{112}\lambda_{132,312,231,321} < 10^{-16}$	$p \to K^+ + e^\pm + \mu^\mp + \nu$

Table 1: Upper bounds on various $\lambda_{ijk}, \lambda'_{lmn}$ and λ''_{opq} R-violating couplings from rare processes [6]. Limits on the products of pairs of couplings are scaled by $(\tilde m/(100\,\mathrm{GeV}))^2$, where $\tilde m$ is the mass of the relevant supersymmetric particle exchanged.

from the superpotential, whilst the Higgs-mixing μ term

$$\mu h_u h_d\,, \tag{1.4}$$

has even R parity, and is allowed among the Yukawa couplings in the MSSM lagrangian.[1]

Another possibility, however, is that R violation does occur at some level, which would have interesting phenomenological implications. Some at least of the couplings in (1.2) could be non zero, although they should be adequately suppressed. Stringent bounds on products of couplings arise from various exotic reactions, some of them presented for convenience in table 1.

In order to obtain a consistent extension of the low-energy phenomenological lagrangian, the MSSM gauge group $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ should be embedded in a

[1]The existence of this term is phenomenologically necessary, with the mass parameter μ being of the order of the electroweak scale. This is potentially an issue, since in many models the μ term is generated dynamically via the vev of some scalar field(s), and one must ask whether the electroweak scale is a natural order of magnitude.

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higher symmetry. Non-abelian GUT groups such as SO(10) and SU(5) provide some successful relations between the various Yukawa couplings [7], but cannot eliminate all the unwanted terms in (1.2). An important role in resolving these issues can be played by additional U(1) symmetries, which were originally used to solve the fermion mass hierarchy problem [8].

One consistent picture which may give a final answer to these questions has emerged in effective field theory models derived from strings. In these constructions, one usually encounters a unified or partially unified non-abelian gauge symmetry, accompanied by a number of external U(1) factors, which may play the role of family symmetries.[2] The fermion mass textures and other Yukawa interactions of the above models are dictated by the particular charges of the particles under the U(1) symmetries, the specific flat direction which has been chosen, and further string selection rules and other string symmetries. Several attempts have had successful predictions: to our knowledge, the most promising results have been obtained in the context of the free-fermionic formulation of the heterotic superstring [9]. To construct a model, one has first to determine a suitable set of boundary conditions on the worldsheet fermionic degrees of freedom introduced to cancel the conformal anomaly. In this context, all properties of the effective field theory model, i.e. the gauge symmetry group including external U(1) factors, other continuous or discrete symmetries, the Yukawa interactions and their strengths are in principle determined once the flat directions are specified.

Although this approach looks very promising, there are shortcomings, which have mainly to do with the freedom of choice in the string vacuum. There are many possible choices for the sets of boundary conditions on the basis vectors, and each one of them leads to a different group structure. Moreover, even if the gauge group is specified by some other considerations, there are still many basis choices that predict different massless spectra and Yukawa interactions. A third source of ambiguity arises from the non uniqueness of the vacuum, associated with different flat directions of the effective potential. These arise because a string model typically contains, in addition to the MSSM spectrum, a number of singlet fields with zero electric charge. These singlets may acquire non-zero vevs and provide masses to various fermions and other particles through their tree-level and non-renormalizable Yukawa couplings. Their vevs have to respect certain constraints necessary to ensure the D- and F-flatness of the superpotential. In general, there are multiple solutions of these flatness conditions, and one has to choose the most appropriate of them guided by phenomenological criteria. In other words, even within the class of models sharing a given string basis, there are multiple string vacua, and it is not known *a priori* which is the most suitable.

It is interesting to note, however, that most of the known string models share several generic properties [10, 11, 12] which point to an one-particular generalization

[2]See, for example, [10, 11, 12] and references therein.

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of the MSSM. Two of the most important are the existence (i) of an anomalous $U(1)_A$ symmetry and (ii) various singlets capable of obtaining non-zero vevs, whose magnitude is fixed once the string vacuum is chosen. Based on the above observations, during the last few years there has developed a new strategy for attacking the problems of fermion masses and other Yukawa interactions. Extensions of the MSSM with new abelian or discrete symmetries [8, 13, 14, 15, 16, 17, 18, 20, 21, 22] have given some insights into the fermion mass hierarchy puzzle thanks to anomalous or non-anomalous U(1) symmetries which distinguish various families. It has been established that, in the MSSM, the existence of one non-anomalous U(1) family symmetry is sufficient to obtain a viable hierarchical fermion mass pattern. Additional non-anomalous U(1) symmetries may also be possible, resulting in further restrictions on the Yukawa lagrangian. Interestingly, the appearance of multiple U(1) symmetries is generic in models derived from the superstring.

In this work, we adopt a slightly different point of view, and go beyond the simple $U(1)_A$ extensions of the MSSM. As described above, another important ingredient of string-derived models is the existence of some intermediate gauge. symmetry,[3] instead of the Standard Model one. As found previously, in most cases this intermediate gauge symmetry provides successful relations between the Yukawa couplings. Moreover, it is known that there are only a few types of such partially-unified groups which dispense with the use of Higgs fields in the adjoint representation [24] to break down to the Standard Model symmetry. Here we provide a general discussion of anomalous $U(1)_A$ family symmetries within the context of the flipped $SU(5)\times U(1)'$ gauge group. Our *first* aim is to determine the possible $U(1)_A$ family symmetries which predict fermion mass matrices consistent with the low-energy measurements. This and the specific non-abelian structure of the theory impose conditions on the $U(1)_A$ charges of the matter and Higgs representations. At a *second* stage, we explore the role of the $U(1)_A$ symmetries in constraining R-violating interactions, baryon-number-violating operators, and Higgs mixing terms. These constraints further reduce the acceptable choices of $U(1)_A$ charges, and therefore the possible fermion mass textures. *Third*, an interesting feature of flipped $SU(5) \times U(1)'$, as opposed to the minimal SU(5) GUT, is that the neutrinos obtain naturally non-zero masses. In the type of model discussed here, we find very restrictive forms for the neutrino mass matrices.

Section 2 is devoted to the presentation of the minimal supersymmetric version of flipped $SU(5) \times U(1)'$. In section 3 we derive the possible forms of R-, B- and L-violating operators in $SU(5) \times U(1)'$, and relate them to the λ, λ' and λ'' parameters of (1.2). In section 4 we discuss the anomaly-cancellation conditions in conjunction with the canonical $\sin^2\theta_W$ condition at the unification scale. The charged fermion mass textures and the general solutions of the anomaly-cancellation conditions determining the $U(1)_A$ charges are given in section 5. Section 6 deals with the neutrino

[3] For earlier attempts, see also [23, 22].

$\mathrm{SU}(5)\times\mathrm{U}(1)$	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$
$F_i(10,1/2)$	$Q(3,2,1/6)+d^c(\bar{3},1,1/3)+\nu^c(1,1,0)$
$\bar{f}_i(\bar{5},-3/2)$	$u^c(\bar{3},1,-2/3)+\ell(1,2,-1/2)$
$e_i^c(1,5/2)$	$e^c(1,1,1)$
$H(10,1/2)$	$Q_H(3,2,1/6)+d_H^c(\bar{3},1,1/3)+\nu_H^c(1,1,0)$
$\bar{H}(\overline{10},-1/2)$	$\bar{Q}_H(3,2,-1/6)+\bar{d}_H^c(\bar{3},1,-1/3)+\bar{\nu}_H^c(1,1,0)$
$h(5,-1)$	$D(\bar{3},1,-1/3)+h_d(2,1,-1/2)$
$\bar{h}\,(\bar{5},1)$	$\bar{D}(\bar{3},1,1/3)+h_u(2,1,1/2)$

Table 2: Decomposition of the $\mathrm{SU}(5)\times\mathrm{U}(1)'$ matter and Higgs into representations of the Standard Model gauge group.

masses and the B-, L- and R-violating couplings in the presence of the anomalous $\mathrm{U}(1)_A$ symmetry. Specific detailed models are presented and discused in section 7, and future extensions and prospects are presented in section 8.

2. Review of the flipped $\mathrm{SU}(5)\times\mathrm{U}(1)'$ model

We present in this section basic features of the supersymmetric version of the flipped $\mathrm{SU}(5)\times\mathrm{U}(1)'$ model. For later purposes and for comparison with the Standard Model charge assignments, it is useful to recall the decomposition of the $\mathrm{SU}(5)\times\mathrm{U}(1)'$ representations in terms of the Standard Model spectrum, which are presented in table 2. This is the minimal version, whose field content includes the MSSM spectrum augmented only by right-handed neutrinos and just two pairs of colored triplets, (d_H^c, D) and $(\bar{d}_H^c, \bar{D})$, which acquire high masses. The superpotential of the minimal model contains the following Yukawa couplings:

$$\begin{aligned}\mathcal{W} = {} & \lambda_u F\bar{f}\bar{h} + \lambda_d FFh + \lambda_e \ell^c \bar{f} h + \\ & + \lambda_\nu F\bar{H}\phi_i + HHh + \bar{H}\bar{H}\bar{h} + \phi_i\phi_j\phi_k + h\bar{h}\phi\,. \end{aligned} \tag{2.1}$$

The first line gives masses to the charged fermion fields of the Standard Model, plus Dirac masses for the neutrinos. Four singlet fields, ϕ and $\phi_{1,2,3}$ are also introduced, but only one of them develops a non-zero vev: $\langle\phi\rangle \approx m_W$. The coupling $\lambda_\nu F\bar{H}\phi_i$ together with the Dirac mass terms obtained from the $F\bar{f}\bar{h}$ term realizes an extended see-saw mechanism which suppresses the left-handed neutrino masses [25]. The term ϕhh, on the other hand, provides an acceptable Higgs mixing term at the electroweak scale.

If the symmetry of the model were just the one described above, however, unwanted terms could also be present. As a first example, we note that the singlet ϕ

may form a mass term $\langle\phi\rangle F\bar{H}$. In addition, the $SU(5)\times U(1)'$ gauge symmetry also allows trilinear terms

$$FHh + H\bar{f}\bar{h}\,, \tag{2.2}$$

which give unacceptable mixings of ordinary fermions with the colour-triplet fields and the Higgs multiplets. In order to avoid these couplings, it is necessary to impose a symmetry beyond $SU(5)\times U(1)'$. The simplest possibility is to assume the Z_2 parity $H\to -H$. This eliminates the terms (2.1), but cannot prevent the mixing term $F\bar{H}\phi$. One may invent another discrete symmetry to prevent this term, while leaving other useful terms untouched, or U(1) symmetries may be added to solve this problem. In the present approach, as we have already discussed in the introduction, we would like to explore the possibility whether a single $U(1)_A$ anomalous symmetry may answer all the above questions in the context of the flipped $SU(5)\times U(1)'$ model. To this end, in the rest of this section we describe our procedure, and formulate the basic questions we think such a symmetry should answer.

• Our first concern is the fermion mass textures. To this end, we first study the constraints on the U(1) charges. In the MSSM case, in all fermion mass terms, the left- and right-handed fields belong to different representations. Thus, the only constraints on them arise from the anomaly-cancellation conditions. In order to have an elegant structure, one may further demand symmetric structures, but this demand restricts considerably the acceptable fermion mass textures. Later, in the final section of this paper, we make some comments about this restriction in the present class of models.

In the case of flipped $SU(5)\times U(1)'$, additional constraints should be taken into account. The left- and right-handed down quarks belong to the *same* representation, namely a decuplet of SU(5), whilst the lepton doublet is in the same pentaplet as the right-handed up quark.

There is another interesting feature of models with a unified or partially unified gauge group. We recall first that the fermion mass textures in the simple U(1) extensions of the MSSM are obtained with only one singlet field, plus its conjugate, via the following non-renormalizable terms:

$$\left(\frac{\langle\phi\rangle}{M_U}\right)^n h\, q_i\, q_j\,, \qquad \left(\frac{\langle\bar{\phi}\rangle}{M_U}\right)^n h\, q_i\, q_j\,, \tag{2.3}$$

where M_U is the unification scale and q_i denotes a fermion field. In the case of models with a larger gauge group, there are Higgs fields $H,\bar{H}$ needed to break the symmetry down to that of the Standard Model. These Higgses may form an effective singlet combination $H\bar{H}$, which can generate additional non-renormalizable contributions to the fermion mass matrices. This creates a new hierarchical structure, in addition to the one obtained from the singlets:

$$\left(\frac{\langle H\bar{H}\rangle}{M_U}\right)^m h\, q_i\, q_j\,. \tag{2.4}$$

The dominant term depends on the choice of the $H, \bar{H}$ charges, and is also restricted by the following very important issue.

• We need to deal with the extra triplets from the pentaplets h and $\bar{h}$. These should receive masses from the terms $HHh + \bar{H}\bar{H}\bar{h}$, which combine them with the uneaten components of $H, \bar{H}$, either at the renormalizable trilinear level or from some higher-order non-renormalizable term. We have the freedom to select the anomalous U(1) charges of $H, \bar{H}$ so that these terms survive the symmetry.

• In the old supersymmetric version of SU(5) × U(1)′ [25], a singlet ϕ_0 with vev $\sim m_W$ was introduced to deal with the Higgs mixing problem. This adds more complications to the hierarchy problem. We have learned from string theory that extra singlet vevs are determined through the anomaly-cancellation condition and are naturally of order $10^{-1}M_{\text{string}}$, though this may be avoided by an acute choice of superpotential. An alternative to a single singlet is to choose the anomalous U(1) charge assignments so that the Higgs mixing term first appears as a high-order non-renormalizable term, in which case a naturally small scale may appear:

$$\mathcal{W}_{h\bar{h}} = \left(\frac{\langle\phi\rangle}{M_U}\right)^r M_U h\bar{h}\,, \tag{2.5}$$

gives a mixing of the right order for $r > 12$ to 15, with the precise value depending on the details of the model, which fix the exact value of the ratio ϕ/M. We should note that this is one of the most restrictive conditions on the anomalous $\mathrm{U}(1)_A$ charges. It will turn out that, in several cases, this restriction results in a rather peculiar charge assignment.

3. The origin of R-, B- and L-violating couplings in flipped SU(5) × U(1)′

In order to explore the constraints imposed on the SU(5) × U(1)′ couplings by an abelian non-anomalous symmetry $\mathrm{U}(1)_A$, we first need to identify the operators (1.2) in the SU(5) × U(1)′-invariant Yukawa superpotential. Thus, we first search for possible higher-order non-renormalizable gauge-invariant terms leading to the R-violating couplings λ, λ' and λ''.

The basic tree-level terms involve, in addition to the fermion mass terms, only couplings of light with heavy states:

$$FFh \longrightarrow qd^c h_d + d^c\nu^c D + qqD\,, \tag{3.1}$$

$$F\bar{f}\bar{h} \longrightarrow qu^c h_u + \ell\nu^c h_u + q\bar{D}\ell + d^c u^c\bar{D}\,, \tag{3.2}$$

$$\bar{f}h\,\ell^c \longrightarrow \ell e^c h_d\,. \tag{3.3}$$

However, protons can decay through the effective dimension-5 operator formed from the combination of the couplings qqD and $d^c u^c\bar{D}$, provided triplet mixing occurs.

The invariant couplings providing masses to the triplets D and $\bar{D}$ are:

$$HHh + \bar{H}\bar{H}\bar{h}\,, \tag{3.4}$$

and possibly the bilinear combinations:

$$\mu_H H\bar{H} + \mu h\bar{h}\,. \tag{3.5}$$

It is important to note that, in the minimal version of $\mathrm{SU}(5)\times\mathrm{U}(1)'$, one triplet pair can be relatively light. The reason [26] is that the dimension-5 operators involving a pair of triplets are proportional not only to the inverse of the mass, but also to the cofactor of the corresponding element of the triplet mass matrix. In the case of minimal $\mathrm{SU}(5)\times\mathrm{U}(1)$, this means that proton decay is proportional to the coefficient of the $H\bar{H}$ term μ_H. If this term is absent or suppressed, as is also required by the F-flatness conditions, the associated operator is also suppressed, independently from the mass of the triplets, which can be quite different.

In addition to the above tree-level terms, the $\mathrm{SU}(5)\times\mathrm{U}(1)'$ symmetry also allows trilinear couplings mixing coloured triplets and Higgs doublets with fermion fields, of the forms:

$$FHh \longrightarrow \langle\nu^c_H\rangle d^c D + h_d\, q d^c_H\,, \tag{3.6}$$

$$H\bar{f}\bar{h} \longrightarrow \langle\nu^c_H\rangle \ell h_u\,, \tag{3.7}$$

The coupling (3.6) induces a large mixing of d^c with D, whilst the term (3.7) gives unacceptably large Higgsino-lepton mixing. An obvious discrete symmetry which may be introduced to prevent these terms without affecting the terms in (2.1) is $H \to -H$.

We recall the absence of renormalizable R-violating terms due to the GUT symmetry. To identify higher-order R-violating terms, we search for all possible invariant non-renormalizable couplings. At lowest (fourth) order, we find

$$10_{1/2}\cdot 10_{1/2}\cdot 10'_{1/2}\cdot \bar{5}_{-3/2}\,, \tag{3.8}$$

with two possible ways to make the contraction, *either*

$$[10_{1/2}\cdot 10_{1/2}][10'_{1/2}\cdot \bar{5}_{-3/2}] \qquad or \qquad [10_{1/2}\cdot 10'_{1/2}][10_{1/2}\cdot \bar{5}_{-3/2}]\,. \tag{3.9}$$

Two of the decuplets contain conventional families F, and one is related to the $\mathrm{SU}(5)$-breaking Higgs H which, at lowest order, gives the following couplings:

$$\begin{aligned}\frac{H}{M}FF\bar{f} &\longrightarrow \frac{\langle\nu^c_H\rangle}{M}(\ell q d^c, u^c d^c d^c)\,, \\ &\longrightarrow \lambda',\, \lambda''\,,\end{aligned} \tag{3.10}$$

where M is some high (unification or string) scale. The second term is

$$\underline{10}'_{1/2}\cdot\bar{5}_{-3/2}\cdot\bar{5}_{-3/2}\cdot 1_{5/2}\,, \tag{3.11}$$

which corresponds to the fields

$$\frac{H}{M}\bar{f}\bar{f}\ell^c \longrightarrow \frac{\langle\nu^c_H\rangle}{M}\ell\,\ell e^c \longrightarrow \lambda\,. \tag{3.12}$$

At higher orders, we find the following dimension-5 operators which violate baryon and lepton number:

$$\frac{\lambda_4^{ijkl}}{M_U}\,F_iF_jF_k\bar{f}_l\,,\qquad \frac{\lambda_5^{ijkl}}{M_U}F_i\bar{f}_j\bar{f}_k\ell^c_l\,, \tag{3.13}$$

where the indices $i,j,k,l=1,2,3$ refer to the three generations. In $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ notation, these are written

$$\frac{\lambda_4^{ijkl}}{M_U}\,q_i\,q_j\,q_k\,\ell_l\,,\qquad \frac{\lambda_5^{ijkl}}{M_U}\,u^c_i\,u^c_j\,d^c_k\,e^c_l\,. \tag{3.14}$$

Although the induced amplitudes of dimension-5 operators are suppressed[4] compared to those arising from products of terms of the forms (3.1) and (3.2), due to the fact that they arise as non-renormalizable interactions scaled by some high mass scale, the baryon-decay bounds on their generalized Yukawa coupling constants are very restrictive. In general, one has to impose $\lambda_4 < 10^{-7}$ for operators involving light quarks, whilst the constraints are less important for λ_5.

4. Anomaly cancellation and fermion mass textures in flipped $\mathrm{SU}(5)\times\mathrm{U}(1)'$

In this section we explore the options in an $\mathrm{SU}(5)\times\mathrm{U}(1)'$ gauge theory with representations charged under an anomalous $\mathrm{U}(1)_A$ symmetry. After finding successful charge assignments, and requiring $\mathrm{SU}(5)\times\mathrm{U}(1)'$ invariance, we find the orders where various mass terms appear in the Yukawa lagrangian.

Our main concern here is the imposition of the anomaly-cancellation conditions. According to [13], in order to obtain the canonical $\sin^2\theta_W$ value at the unification scale, certain conditions should be satisfied by the mixed anomalies of $\mathrm{U}(1)_A$ with the gauge-group factors. In the case of the Standard Model, we have three gauge groups and therefore the Green-Schwarz [27] mechanism combined with the $\sin^2\theta_W$ condition imposes two relations $A_2/A_1 = A_3/A_1 = 5/3$. In the present case, since the effective field theory model resulting from string is based on a higher gauge symmetry,

[4]We note that if there were universality for all couplings λ_4^{ijkl} and λ_5^{ijkl}, the sum of all such operators would vanish for symmetry reasons. However, there is no general argument that such a universality assumption holds in string theory.

we should apply the anomaly-cancellation conditions at this level of symmetry. Thus the $\mathrm{U}(1)_A$ charges should be assigned directly at the GUT level, taking into account the constraints imposed by the particular gauge symmetry.

Assume that the states have the generic $\mathrm{U}(1)_A$ charge assignments shown in table 3. We note that the down-quark mass matrix is automatically symmetric. Concerning the up-quark and charged-lepton mass matrices, we have the freedom to choose either symmetric or non-symmetric textures. We start with the simplest case of symmetric textures.

State	Charge
$F_i(10,1/2)$	α_i
$\bar{f}_i(\bar{5},-3/2)$	β_i
$e_i^c(1,5/2)$	γ_i
$H(10,1/2)$	δ
$\bar{H}\left(\overline{10},-1/2\right)$	$\bar{\delta}$
$h(5,-1)$	ϵ
$\bar{h}\,(\bar{5},1)$	$\bar{\epsilon}$

Table 3: Generic charge assignments under $\mathrm{U}(1)_A$.

In this case, we obtain

$$\alpha_i + \beta_j = \alpha_j + \beta_i \,, \tag{4.1}$$

$$\beta_i + \gamma_j = \gamma_j + \beta_i \,. \tag{4.2}$$

We further require that the third-generation $FFh, F\bar{f}\bar{h}, \bar{f}e^c h$ couplings appear in the renormalizable trilinear superpotential, which implies the further constraints

$$\begin{aligned} \alpha_3 + \beta_3 + \bar{\epsilon} &= 0 \,, \\ 2\alpha_3 + \epsilon &= 0 \,, \\ \beta_3 + \gamma_3 + \epsilon &= 0 \,. \end{aligned} \tag{4.3}$$

Equations (3.8)–(3.10) can be used to solve for $\beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3$ and ε in terms of $\alpha_1, \alpha_2, \alpha_3, \bar{\varepsilon}$. After imposing the symmetric (4.1) and (4.2) and trilinear constraints (4.3), the charge matrix of the up, down quark and lepton masses takes the form

$$C^{FFh} = C^{F\overline{f}\overline{h}} = C^{\overline{f}e^c h} = \begin{pmatrix} 2(\alpha_1-\alpha_3) & (\alpha_1-\alpha_3)+(\alpha_2-\alpha_3) & \alpha_1-\alpha_3 \\ (\alpha_1-\alpha_3)+(\alpha_2-\alpha_3) & 2(\alpha_2-\alpha_3) & \alpha_2-\alpha_3 \\ (\alpha_1-\alpha_3) & (\alpha_2-\alpha_3) & 0 \end{pmatrix} \tag{4.4}$$

In order to have acceptable quark masses, we must impose

$$\alpha_1 - \alpha_3 = \frac{n}{2}\,, \qquad \alpha_2 - \alpha_3 = \frac{m}{2}\,, \quad \text{where} \quad m+n \neq 0,\; m,n = \pm 1, \pm 2, \ldots . \tag{4.5}$$

The resulting effective field theory model has to be anomaly-free, and the introduction of an extra $\mathrm{U}(1)_A$ group factor leads to anomalies which should be cancelled. The Green-Schwarz anomaly-cancellation mechanism may cancel the pure $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_A$-gravitational anomalies, but there are mixed anomalies of the form $A_i = (G_i G_i\,\mathrm{U}(1)_A)$, where $G_i = (\mathrm{SU}(5), \mathrm{U}(1)_Y)$. In terms of the $\mathrm{U}(1)_A$ charges, in the case of $\mathrm{SU}(5)\times\mathrm{U}(1)$ these are written as

$$A_5 : \sum \alpha_i + \frac{1}{2}\sum \beta_i + \left(\delta + \bar{\delta}\right) + \frac{1}{2}(\epsilon + \bar{\epsilon})\,, \tag{4.6}$$

$$A_1 : \frac{5}{2}\sum \alpha_i + \frac{45}{4}\sum \beta_i + \frac{25}{4}\sum \gamma_i + \frac{5}{2}\left(\delta + \bar{\delta}\right) + 5(\epsilon + \bar{\epsilon}) \,, \tag{4.7}$$

$$A_0 : 5\sum \alpha_i^2 - \frac{15}{2}\sum \beta_i^2 + \frac{5}{2}\sum \gamma_i^2 + 5\left(\delta^2 - \bar{\delta}^2\right) + 5\left(\bar{\epsilon}^2 - \epsilon^2\right) = 0 \,. \tag{4.8}$$

In the case of flipped $\mathrm{SU}(5) \times \mathrm{U}(1)'$, gauge-coupling unification imposes the following relation between the gauge couplings:

$$\frac{25}{\alpha_Y} = \frac{24}{\alpha_1} + \frac{1}{\alpha_5} \,. \tag{4.9}$$

Then, the standard value

$$\sin^2\theta_W = \frac{3}{8} \,, \tag{4.10}$$

at the unification scale leads to the following condition for the anomalies:

$$\frac{A_1}{A_5} = 10 \,. \tag{4.11}$$

In order to handle the above system of equations, we find it convenient to change to a new set of variables. We define $\delta_\pm = \delta \pm \bar{\delta}$ and express the α_1, α_2 charges in the anomaly condition equations in terms of the integers m, n from definitions (4.5). Then, condition (4.11) can be solved in terms of δ_+:

$$\delta_+ = \frac{m+n}{3} + 2\alpha_3 \,. \tag{4.12}$$

Substituting in the quadratic constraint (4.8) and solving for $\delta_- = \delta - \bar{\delta}$, we obtain the expression

$$\delta_- = -\frac{(\bar{\epsilon} + 2\alpha_3)(m + n + 2\alpha_3 - \bar{\epsilon})}{(m+n)/6 + \alpha_3} \,, \qquad \alpha_3 \neq -\frac{m+n}{6} \,. \tag{4.13}$$

In this general solution of the anomaly constraint equations, we have expressed all the $\mathrm{U}(1)_A$ charges of the various fields in terms of four parameters, namely the integers m, n and the charges α_3 and $\bar{\epsilon}$. Alternatively, we may use (4.13) to express one of $\alpha_3, \bar{\epsilon}$ in terms of δ_-. In table 4, we give all field charges in terms of the above free parameters.

The above solution is valid as long as the denominator of the expression defining δ_- in (4.13) is non zero, i.e. whenever $\alpha_3 \neq -(m+n)/6$. In the particular case where $\alpha_3 = -(m+n)/6$, we have no constraint for δ_-, but (4.8) gives the following two solutions for $\bar{\epsilon}$: either

$$\bar{\epsilon} = \frac{m+n}{3} \tag{4.14}$$

or

$$\bar{\epsilon} = \frac{2(m+n)}{3} \,. \tag{4.15}$$

Solution (4.15) must, however, be rejected in the case of an anomalous $\mathrm{U}(1)$, since it leads to $A_5 = A_1 = 0$. The $\mathrm{U}(1)_A$ charges of the acceptable solution are presented in table 5.

field	generation		
	1	2	3
F	$n/2+\alpha_3$	$m/2+\alpha_3$	α_3
$\bar{f}$	$n/2-\bar{\varepsilon}-\alpha_3$	$m/2-\bar{\varepsilon}-\alpha_3$	$-\bar{\varepsilon}-\alpha_3$
ℓ^c	$n/2+\bar{\varepsilon}+3\alpha_3$	$m/2+\bar{\varepsilon}+3\alpha_3$	$\bar{\varepsilon}+3\alpha_3$
Higgs			
H	δ	$\bar{H}$	$\bar{\delta}$
h	$-2\alpha_3$	$\bar{h}$	$\bar{\varepsilon}$

Table 4: The $\mathrm{U}(1)_A$ charge assignments in solution (4.13) for symmetric mass matrices in flipped $\mathrm{SU}(5)\times\mathrm{U}(1)'$. The variables $\delta, \bar{\delta}$ are not free parameters, but can be expressed in terms of $\alpha_3, \bar{\varepsilon}$ (see relations (4.12) and (4.13)).

field	generation		
	1	2	3
F	$2n-m/6$	$2m-n/6$	$-(m+n)/6$
$\bar{f}$	$(2n-m)/6$	$(2m-n)/6$	$-(m+n)/6$
ℓ^c	$(2n-m)/6$	$(2m-n)/6$	$-(m+n)/6$
Higgs			
H	δ	$\bar{H}$	$-\delta$
h	$(m+n)/3$	$\bar{h}$	$(m+n)/3$

Table 5: The $\mathrm{U}(1)_A$ charge assignments in solution (4.14) for symmetric mass matrices in flipped $\mathrm{SU}(5)\times\mathrm{U}(1)'$.

5. Charged-fermion mass matrices

In the previous section, we have shown that consistency with the anomaly-cancellation conditions for the $\mathrm{U}(1)_A$ charges leads to two distinct solutions. It is interesting to observe that the mass matrices depend in both solutions only on the integer parameters m, n. The charge matrices now have a universal form

$$C^{FFh} = C^{F\overline{fh}} = C^{\overline{f}e^ch} = \begin{pmatrix} n & \frac{m+n}{2} & \frac{n}{2} \\ \frac{m+n}{2} & m & \frac{m}{2} \\ \frac{n}{2} & \frac{m}{2} & 0 \end{pmatrix}. \tag{5.1}$$

It is important to note here that the quark and lepton mass matrices have *identical* powers of non-renormalizable terms in all entries, in contrast to the case of the MSSM [21]. In the MSSM, the quark textures are also expressed in terms of two parameters m, n as above, but the charged-lepton mass matrix is given by

$$C^{\ell e^c h}_{\mathrm{MSSM}} = \begin{pmatrix} n & \frac{m+n}{2} & \frac{n}{2} \\ \frac{m+n}{2} & m+p & \frac{m+p}{2} \\ \frac{n}{2} & \frac{m+p}{2} & 0 \end{pmatrix}, \tag{5.2}$$

where p is another integer. Clearly, the higher GUT symmetry in the present model is more restrictive than in the case of the MSSM, and, as a result, this parameter is forced to be zero: $p = 0$.

The actual form of the mass matrices is obtained by legal non-renormalizable terms involving powers of singlets which cancel the charge of the corresponding entry in (5.1). As mentioned above, there is a significant difference in this regard, as compared to the MSSM case. In the MSSM, in the minimal case one introduces a pair of singlet fields $\phi, \bar{\phi}$ which obtain vevs and contribute to the mass matrix entries through two dimensionless parameters $\lambda = \phi/M_U, \bar{\lambda} = \bar{\phi}/M_U$. In the present case, in addition to $\phi, \bar{\phi}$ there is an effective singlet formed by the combination $\kappa = H\bar{H}/M_U^2$ of the Higgs representations breaking the $\mathrm{SU}(5)\times\mathrm{U}(1)$ symmetry to that of the SM. Therefore, the structure of the fermion mass textures is much richer in this case. There are now three expansion parameters entering in the mass matrices, namely $\lambda, \bar{\lambda}$ and κ. All of them have vevs with well-defined magnitudes, as the first two are related to the D-term cancellation, whilst the third one determines the $\mathrm{SU}(5)\times\mathrm{U}(1)'$ breaking scale. Let q_i, q_j^c denote fermions of the generations i, j respectively, and h the Higgs which couples to these fields. Then, a particular mass entry with $\mathrm{U}(1)_A$ charge C_{ij} has in general the form

$$\kappa^x \lambda^y \bar{\lambda}^z q_i\, q_j^c\, h \tag{5.3}$$

which implies the following condition on the related charges:

$$C_{ij} + x\delta_+ + y q_\phi + z\bar{q}_\phi = 0\,, \tag{5.4}$$

where x, y, z are integers representing the powers of the three singlets required to make the term $\mathrm{U}(1)_A$-invariant, and $q_\phi, \bar{q}_\phi$ the $\mathrm{U}(1)_A$ charges of the singlets. Without loss of generality, we may assume that $q_\phi = -\bar{q}_\phi = 1$, so the above relation can simply be written as $C_{ij} + x\delta_+ + \omega = 0$ with $\omega = y - z$. The largest contribution arises when either of y, z are zero, so we may write the non-renormalizable term formally as follows:

$$\kappa^x \rho^{|\omega|} q_i\, q_j^c\, h\,, \tag{5.5}$$

where

$$\rho = \lambda\theta(\omega) + \bar{\lambda}\theta(-\omega)\,.$$

Before we analyze the mass matrices, it is useful to make a rough estimate of the orders of magnitude of the above parameters. The parameter κ is related to the

$SU(5) \times U(1)'$ breaking scale M_5. In the most common scenario, this scale is close to the MSSM unification scale, which is about two orders of magnitude below the string scale. The parameters $\lambda, \bar{\lambda}$ are related to the anomalous $U(1)_A$ breaking scale. It is important to note here that one is forced to take $\lambda \neq \bar{\lambda}$ since, in the case of an anomalous $U(1)_A$ symmetry, we should necessarily take $\phi \neq \bar{\phi}$ in order to cancel the D term. In particular, the Green-Schwarz anomaly-cancellation mechanism generates a constant Fayet-Iliopoulos [28] contribution to the D term of the anomalous $U(1)_A$. This is proportional to the trace of the anomalous charge over all fields capable of obtaining non-zero vevs. To preserve supersymmetry, the following D-flatness condition should be satisfied [29, 30]:

$$\sum_{\phi_j} Q_j^X |\phi_j|^2 = -\xi \neq 0 \,, \tag{5.6}$$

where Q_j^X is the $U(1)_A$ charge of the singlet ϕ_j. The parameter ξ depends on the common gauge coupling at M_{string}, and on M_{string} itself. In the present case, where only two singlet fields $\phi, \bar{\phi}$ are involved, this condition becomes

$$|\langle\bar{\phi}\rangle|^2 - |\langle\phi\rangle|^2 = \xi \,. \tag{5.7}$$

The scales of the ϕ_j and $\bar{\phi}_j$ vevs are naturally about one order of magnitude smaller than M_{string}. Therefore, in a natural scenario for the hierarchy of scales, we expect

$$\langle H\rangle \approx \mathcal{O}\left(10^{-1}\right)\langle\phi\rangle \approx \mathcal{O}\left(10^{-2}\right) M_{\mathrm{string}} \tag{5.8}$$

and subsequently $\kappa < \bar{\lambda} < 1$. As can be seen from (5.7), the remaining parameter λ is not completely determined: due to the positivity of ξ, we need only impose $\lambda < \bar{\lambda}$.

Up to now, we have imposed the gauge-invariance constraints on the fermion mass entries in a manner analogous to the MSSM case. In the present case, however, there are additional constraints related to the appearance of the extra colour-triplet states contained in $H, \bar{H}$ and $h, \bar{h}$ representations, which should be massive at some high scale. Indeed, triplet-doublet splitting requires the existence of the couplings HHh and $\bar{H}\bar{H}\bar{h}$. If we want these couplings to be present at some order k and $\bar{k}$ respectively, we have to impose two extra constraints:

$$2\delta + \epsilon = k \,, \qquad 2\bar{\delta} + \bar{\epsilon} = \bar{k} \,, \tag{5.9}$$

where $k, \bar{k}$ should be integers. We can now exchange the parameters $\bar{\epsilon}, \alpha_3$ of solution (1.4) for the integers $k, \bar{k}$:

$$\bar{\varepsilon} = \frac{\left(9k^2 + 6\bar{k}^2 - 3\bar{k}(m+n) - (m+n)^2 + k(15\bar{k} - 4(m+n)\right)}{3\left(5k + 4\bar{k} - 3(m+n)\right)} \,,$$

$$\alpha_3 = \frac{\left(6k^2 + 6\bar{k}^2 - 14\bar{k}(m+n) + 7((m+n)^2 + 3k(4\bar{k} - 5(m+n)\right)}{6\left(5k + 4\bar{k} - 3(m+n)\right)} \tag{5.10}$$

for $(5k+4\bar{k}-3(m+n)) \neq 0$, while for the solution (4.14) we have

$$\bar{k} = \frac{2(m+n)}{3} - k\,, \qquad \delta_{-} = k - \frac{m+n}{3}\,. \tag{5.11}$$

These are the most general relations which correlate the values of $\bar{\epsilon}$ and α_3 with the integer numbers $m, n, k, \bar{k}$.

The above solutions satisfy all our constraints, but we draw attention to one subtle point. The anomaly A_1 should *not* vanish, precisely because we are postulating an anomalous U(1). Indeed, putting the solutions into (4.7), we get

$$A_1 = 10A_5 = -5\left(k-\bar{k}\right)\frac{k+\bar{k}-m-n}{5k+4\bar{k}-3(m+n)} \tag{5.12}$$

for solution (4.12) and

$$A_1 = 10A_5 = \frac{10(m+n)}{3} \tag{5.13}$$

for solution (4.13).

An important prediction can now be extracted from the last two formulae for the coloured triplet couplings: we observe that the solution (5.12) involves the factor $k-\bar{k}$ in the numerator. This tells us that, when $k=\bar{k}$, the mixed anomalies vanish and the U(1) symmetry must be anomaly-free. Therefore, we infer that in the presence of an anomalous U(1) symmetry it is not possible to obtain both the terms HHh and $\bar{H}\bar{H}\bar{h}$ at the same order. In particular, we conclude that these terms *cannot provide simultaneously* renormalizable trilinear superpotential masses for the two triplet pairs, because it is not possible to have $k=\bar{k}=0$. This should not be considered as a drawback, since, as was argued in section 3, a light triplet pair can be tolerated, because it does not necessarily lead to rapid baryon decay.

It is worth pointing out here that one need not necessarily embed this model in a usual string context. In this case, there is no necessity to impose a non-anomalous U(1) symmetry, and the mixed anomalies may well vanish, provided that the $\mathrm{U}(1)^3$ anomalies also vanish. This may also happen to the present model, provided that we also include additional pairs of states whose contribution to the $\mathrm{U}(1)_A^3$ anomaly exactly cancels the contributions of the standard particles' $\mathrm{U}(1)_A$ charges. In this case, one can find other solutions which lead to a viable low-energy model, as we demonstrate below with a simple choice.

As an example of this particular case, we work out here the simplest example where both couplings appear in the trilinear superpotential.[5] For $k=\bar{k}=0$, there exists only one solution, with $\bar{\epsilon}=(m+n)/9$, $\alpha_3=-7(m+n)/18$ and $\delta_{+}=-\frac{4}{9}(m+n)$. We next seek solutions of (5.4) involving the minimum powers of the expansion parameters $\kappa, \lambda, \bar{\lambda}$. Bearing in mind the relative magnitudes of $\kappa, \bar{\lambda}$ estimated above,

[5]We note, however, that the phenomenology would be more interesting if one of these couplings appears at higher order, resulting in a light colored triplet.

we may derive the possible lowest-order contributions to the mass-matrix entries. The expansion parameter κ has significant contributions only for $x = 0,1,2$, and one finds

$$\begin{array}{llll}
\text{11-entry: } & x=0,\ \omega=-n\,; & x=1,\ \omega=\dfrac{4m-5n}{9}\,; & x=2,\ \omega=\dfrac{8m-n}{9}\,, \\
\text{12-entry: } & x=0,\ \omega=-\dfrac{m+n}{2}\,; & x=1,\ \omega=-\dfrac{m+n}{18}\,; & x=2,\ \omega=\dfrac{7(m+n)}{18}\,, \\
\text{13-entry: } & x=0,\ \omega=-\dfrac{n}{2}\,; & x=1,\ \omega=\dfrac{8m-n}{18}\,; & x=2,\ \omega=\dfrac{16m+7n}{18}\,, \\
\text{22-entry: } & x=0,\ \omega=-m\,; & x=1,\ \omega=\dfrac{4n-5m}{9}\,; & x=2,\ \omega=\dfrac{8n-m}{9}\,, \\
\text{23-entry: } & x=0,\ \omega=-\dfrac{m}{2}\,; & x=1,\ \omega=\dfrac{8n-m}{18}\,; & x=2,\ \omega=\dfrac{7m+16n}{18}\,.
\end{array}$$

We now present a specific example. Starting from the μ term, in order to obtain $\mu \approx \mathcal{O}(m_W)$, we take $\epsilon + \bar{\epsilon} \equiv \frac{8}{9}(m+n) = 16$, which implies $m+n=18$. Taking $m=4, n=14$, as a viable choice, we solve eqs. (5.4) to determine the possible solutions for x, y, z. Some comments are in order here. It is convenient to replace y, z with a new single variable $\omega = y - z$. When a solution gives a positive (negative) value for ω, this means that $\lambda(\bar{\lambda})$ should appear in the operator. Taking into account this and the hierarchy of scales discussed above, the following dominant terms appear:

$$\frac{M^{FFh}}{\langle h \rangle} \approx \frac{M^{F\bar{f}\bar{h}}}{\langle \bar{h} \rangle} \approx \frac{M^{\bar{f}e^c h}}{\langle h \rangle} \approx \begin{pmatrix} \kappa^2\bar{\lambda}^6 & \kappa^2\bar{\lambda} & \kappa^2\lambda \\ \kappa^2\bar{\lambda} & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \kappa^2\lambda & \bar{\lambda}^2 & 1 \end{pmatrix}. \tag{5.14}$$

From (5.7) we may take $\lambda < \bar{\lambda}$, so that the 13-entry becomes small. Since $\kappa < \bar{\lambda}$, we conclude that a correct hierarchical mass pattern can be obtained in a natural way.

We return now to the general case with a non-anomalous U(1) symmetry. It is easy to see that, among the possible $\mathrm{U}(1)_A$ charge assignments, there are numerous possibilities that could lead to a viable fermion mass structure. The construction of a successful effective field theory model is however more complicated, as we have explained in previous sections, since a large number of other phenomenological constraints have to be respected. To start with, we first calculate the charges of the two bilinears $H\bar{H}$ and $h\bar{h}$ in the solutions found above. As has already been noted, they are both important, since the first is directly related to the fermion mass textures, whilst the second is related to the μ term. The charges of the $H\bar{H}$ and $h\bar{h}$ terms are

$$\delta + \bar{\delta} = \frac{2\left(k - \bar{k} - m - n\right)\left(3\left(k+\bar{k}\right) - 2(m+n)\right)}{3\left(5k + 4\bar{k} - 3(m+n)\right)}\,,$$

$$\epsilon + \bar{\epsilon} \equiv r = \frac{3k\left(k+\bar{k}\right) + 11\left(k+\bar{k}\right)(m+n) - 8(m+n)^2}{3(5k+4\bar{k}-3(m+n))} \tag{5.15}$$

for solution (4.12), and

$$\delta + \bar{\delta} = 0\,, \qquad \epsilon + \bar{\epsilon} = \frac{2(m+n)}{3}\,, \tag{5.16}$$

for solution (4.14). The second solution allows for a renormalizable $H\bar{H}\phi$ term, which prevents the breaking of SU(5) × U(1) to the Standard Model. Thus we will elaborate further on solution (4.14).

In order to determine the appropriate $\mathrm{U}(1)_A$ charges of the fields, we scan the above equations for solutions with $\epsilon + \bar{\epsilon}$ = integer ≥ 12 at least. However, the following subtle point arises here when we consider the sum $\delta + \bar{\delta}$: if $\delta + \bar{\delta}$ is not an integer, then $\langle H\bar{H}\rangle$ can fill in non-integer entries in the mass matrices. In this case, the parameters m, n involved may also take non-integer values, and therefore we have to reconsider our analysis. In the present construction we consider only cases which admit integer δ_+ values.

Our next concern is the R-violating terms whose implications were discussed in section 2. To estimate their effects in a particular solution, we need to know at which order they appear. This depends on the $\mathrm{U}(1)_A$ charge each of these operators carries, which has to be cancelled by the appropriate number of singlets. In tables 6 and 7 we present the $\mathrm{U}(1)_A$ charges of the two tree-level couplings $H\bar{f}\bar{h}$ and F_iHh for the two acceptable solutions found above. Bearing in mind the limits from table 1, we infer that, if the above two couplings are to be present in the effective model, then large values of m, n and δ, k are needed in order to suppress them sufficiently. What other possibilities might we have? There are two other ways to avoid conflict with phenomenological limits.

(i) We note again that it is possible to avoid these terms by imposing a discrete symmetry on the Higgs field: $H \to -H$. This choice would have further consequences for other types of operators. For example, the operator $H\bar{H}$ cannot appear with odd powers, unless the discrete symmetry applies to both $H, \bar{H} \to -H, -\bar{H}$. In this case, all even and odd non-renormalizable terms involving κ still remain, and contribute to the mass matrices.

(ii) A second possibility is to consider solutions which predict non-integer values for the net charges of the above operators. In this case, it is not possible to form $\mathrm{U}(1)_A$-invariant non-renormalizable terms, and the operators do not appear in the effective theory. In the next sections we present various solutions where each of the above cases are realized.

6. Neutrino mass textures and proton-decay operators

In this section, we derive the mass textures for the neutral fermion sector, i.e. the Dirac and Majorana mass matrices for the left- and right-handed neutrinos. An

$F_1Hh,\ H\bar{f}_1\bar{h}$	$(k+n)/2$
$F_2Hh,\ H\bar{f}_2\bar{h}$	$(k+m)/2$
$F_3Hh,\ H\bar{f}_3\bar{h}$	$k/2$

Table 6: The $\mathrm{U}(1)_A$ charges of the F_iHh and $H\bar{f}_i\bar{h}$ terms for the solution (1.4).

$F_1Hh,\ H\bar{f}_1\bar{h}$	$\delta+(m+4n)/6$
$F_2Hh,\ H\bar{f}_2\bar{h}$	$\delta+(4m+n)/6$
$F_3Hh,\ H\bar{f}_3\bar{h}$	$\delta+(m+n)/6$

Table 7: The $\mathrm{U}(1)_A$ charges of the F_iHh and $H\bar{f}_i\bar{h}$ terms for solution (4.14).

interesting consequence of the group structure and the anomalous $\mathrm{U}(1)_A$ symmetry that becomes clear immediately is the intimate relation of the neutrino mass matrix structure to dimension-5 proton decay operators. We start with the neutrino physics.

In contrast to the minimal $\mathrm{SU}(5)\times\mathrm{U}(1)'$ model, an inevitable consequence of extending the MSSM to a supersymmetric model with flipped $\mathrm{SU}(5)\times\mathrm{U}(1)'$ symmetry is the appearance of non-zero neutrino masses. It is well known that recent neutrino data strongly suggest a non-zero, although tiny, neutrino mass, so this $\mathrm{SU}(5)\times\mathrm{U}(1)$ prediction is very welcome. The interesting fact in the present approach is that, once we have fixed the $\mathrm{U}(1)_A$ charges in order to rederive the observed charged-fermion mass hierarchy, we make definite predictions for the neutrino mass sector, as we now analyze further. The Dirac neutrino mass matrix is derived from the same couplings as that of the up quarks, so its form is fixed from (4.4). There are two different Majorana mass matrices, one for the left- and another for the right-handed components, which have vastly different mass scales. Direct left-handed Majorana masses may arise from terms involving the combinations $\bar{f}_i\bar{f}_j$. The general gauge invariant form of the relevant terms is

$$m_\nu \bar{f}_i\bar{f}_j = \bar{f}_i\bar{f}_j\langle H\rangle^{10}\frac{\langle hh\rangle}{M}\kappa^{2x}\rho^{|\omega|}\,, \tag{6.1}$$

where κ and ρ are the expansion parameters introduced above, ω is an integer, and x a non-negative integer. In order to compensate the $\mathrm{U}(1)'$ charge, an additional vev $\langle H\rangle^{10}$ is introduced, which results in a big suppression compared to the effective generation of light Majorana masses through the see-saw mechanism. Let us point out here another important difference from the MSSM case. In the MSSM, both terms are of the same order, but in $\mathrm{SU}(5)\times\mathrm{U}(1)'$ the big suppression of the m_ν contribution arises due to the particular charge assignments of the representations under the $\mathrm{U}(1)'$ symmetry. In order to determine the effective light-Majorana mass matrix, we further need the form of the heavy right-handed neutrino mass texture. The gauge-invariant terms contributing to the latter are

$$\mathcal{M}_N F_iF_J = F_iF_j\frac{\bar{H}\bar{H}}{M}\kappa^x\rho^{|\omega|}\,. \tag{6.2}$$

There are also possible higher-order contributions from invariants of the type F_iF_jHHH, but these are expected to be relatively suppressed. Thus, this matrix is also completely specified once we have made a specific choice of the $\mathrm{U}(1)_A$ charges. Then, the effective light neutrino matrix relevant to experimental detection is found to be

$$\begin{aligned} m_\nu^{\mathrm{eff}} &= m_\nu - \frac{1}{4} m_D^T \mathcal{M}_N^{-1} m_D \\ &\approx -\frac{1}{4} m_D^T \mathcal{M}_N^{-1} m_D \,, \end{aligned} \tag{6.3}$$

since, according to the previous comment concerning the $\bar{f}_j \bar{f}_i$ terms, only the second contribution to m_{eff} is relevant here.

Hence, in order to obtain the structure of the effective light Majorana mass matrix, we need to know the structures of the matrices m_D and $\mathcal{M}_N$. However, due to the GUT symmetry, the Dirac mass matrix is already determined and has the same form as the down-quark mass matrix. In order to determine the heavy Majorana matrix (6.2), we need to calculate all possible combinations $C_{ij}^M = F_i F_j (\bar{H})^2$. Another subtle issue arises here, however: if we wish to obtain non-zero heavy Majorana mass entries, the charge matrix entries C_{ij}^M have to be integers, which implies that the charges of the quantities $F_i \bar{H}$ should be either integers or half-integers. In the former case, the bilinears $F_i \bar{H}$, which usually lead to unacceptable mixing, appear in the superpotential. Thus it is desirable to obtain a solution for the $\mathrm{U}(1)_A$ charge assignments which provides half-integer charges for the $F_i \bar{H}$ operators. We also note that a simple way to avoid these terms without projecting out the useful heavy-neutrino contributions is to adopt the discrete symmetry $\bar{H} \to -\bar{H}$ as was also discussed previously.

In what follows, we make a general analysis and present solutions for both cases, without mentioning further any additional discrete symmetry. To express the charges of the $F_i \bar{H}$ and $F_i F_j (\bar{H})^2$ terms, we introduce the parameter $r = \varepsilon + \bar{\varepsilon}$. Then the charges of the $F_i \bar{H}$ terms are

$$\text{charge}\left(F_i \bar{H}\right) = \frac{\bar{k} - r}{2} + \frac{1}{2} v_i \,, \tag{6.4}$$

where v_i is the i-th component of the vector

$$\mathbf{v} = \begin{pmatrix} n \\ m \\ 0 \end{pmatrix} . \tag{6.5}$$

We make here an important remark: as has been explained on several occasions in this work, in order to have a μ term, it is necessary to ensure an integer value for the sum of the doublet Higgs charges $r = \varepsilon + \bar{\varepsilon}$.

The above property has a direct implication for the existence of the heavy Majorana mass matrix. The latter is related to the charge entries

$$\text{charge}\left(F_i F_j (\bar{H})^2\right) = \bar{k} - r + c_{ij} \,, \tag{6.6}$$

which has just the structure of the fermion matrix entries, shifted by the common value $\bar{k}-r$. This implies that the heavy Majorana mass matrix has the same texture, but with its entries are all shifted by the power $\bar{k}-r$. Thus, we end up with a very precise form for the Majorana mass matrix, which gives definite predictions for neutrino physics. If, in particular the scale where the matrix $\mathcal{M}_N$ is formed is related to that of the charged-lepton mass matrix, then, at the unification scale we may write

$$\mathcal{M}_N = \rho^{2|r|}\frac{m_\ell}{\langle h\rangle}M_U\,, \tag{6.7}$$

where m_ℓ is the charged-lepton mass matrix, whose form is dictated by the charge assignment (4.4).

In a similar way, we can examine whether a certain charge assignment allows the existence of dangerous dimension-5 operators: we find that their charges are expressible in terms of the same parameters. The charges of the operators involving only matter fields can be written as follows:

$$\text{charge}\left(F_iF_jF_s\bar{f}_s\right) = \text{charge}\left(F_1\overline{f}_i\overline{f}_j{\ell^c}_s\right) = -r + c_{ij} + v_s\,, \tag{6.8}$$

where c_{ij} is the charge of the (ij) charge fermion entry in (4.4), and v_s is the s component of the vector $\mathbf{v}$ defined in (6.5). As has been shown earlier, the first operator leads to the dangerous $QQQ\ell$ combination which leads to fast proton decay. Operators of this type involving the first two generations should have a rather small effective coupling of the order $\lambda_4 \approx 10^{-6}$ or less. Another set of dangerous B- and L-violating operators arises when we replace one of the matter representations F_i above with the Higgs field H to obtain $F_iF_jH\bar{f}_s$ and $H\bar{f}_i\bar{f}_j\ell^c_s$, respectively. In this case, we find that their charges are simply the above ones shifted by the amount $(k-v_s)/2$:

$$\text{charge}\left(HF_iF_j\bar{f}_s\right) = \text{charge}\left(H\overline{f}_i\overline{f}_j{\ell^c}_s\right) = -r + \frac{k}{2} + c_{ij} + \frac{1}{2}v_s\,. \tag{6.9}$$

It turns out that these operators put rather stringent limits on the $\mathrm{U}(1)_A$ charge assignments. Indeed, as noted earlier in this paper, these give R-violating couplings of the form

$$\lambda''_{ijs}u^c\,d^c\,d^c \approx \left(\frac{\phi}{M}\right)^x\frac{\langle\nu^c_H\rangle}{M}u^cd^cd^c\,, \tag{6.10}$$

where x is an appropriate power. We see from table 1 that the most stringent bound comes from the product of couplings $\lambda'_{112}\,\lambda''_{112}$ which, in the case of flipped SU(5) are equal as can be seen from (3.10). Here the exponent takes the form $x = |n-r+(k+n)/2|$ thus for an expansion parameter $\rho\sim 0.2$, the bound $\lambda'_{112}\lambda''_{112}\sim \lambda'^2_{112} < 10^{-21}$ shown in table 1 requires a rather large value of $x\geq 14$. We also mention that the other two possibilities for avoiding these operators are either to have non-integer charge, or to introduce the R-parity symmetry. Below, we present examples

field	generation		
	1	2	3
F	1	3	5
$\bar{f}$	-1	1	3
ℓ^c	3	5	7
Higgs			
H	1	$\bar{H}$	5
h	-10	$\bar{h}$	-8

Table 8: Solution A: $k = -8, \bar{k} = 2, m = -4, n = -8, r = -18$.

field	generation		
	1	2	3
F	2	0	-2
$\bar{f}$	2	0	-2
ℓ^c	2	0	-2
Higgs			
H	$-3/2$	$\bar{H}$	$3/2$
h	4	$\bar{h}$	4

Table 9: Solution B: $k = 1, \bar{k} = 7, m = 4, n = 8, r = 8$.

field	generation		
	1	2	3
F	$-3/2$	-3	$-9/2$
$\bar{f}$	$3/2$	0	$-3/2$
ℓ^c	$-9/2$	-6	$-15/2$
Higgs			
H	0	$\bar{H}$	-6
h	9	$\bar{h}$	6

Table 10: Solution C: $k = 9, \bar{k} = -6, m = 3, n = 6, r = 15$.

field	generation		
	1	2	3
F	5	-5	-3
$\bar{f}$	8	-2	0
ℓ^c	2	8	-6
Higgs			
H	$-11/2$	$\bar{H}$	$7/2$
h	6	$\bar{h}$	3

Table 11: Solution D: $k = -5, \bar{k} = 10, m = -4, n = 16, r = 9$.

where all there operators carry half-integer charges and therefore do not appear in the Yukawa lagrangian.

We have now all the ingredients needed for specific solutions of the constraints. We present various examples with reasonable charge assignments which lead to different types of low-energy phenomenological models.

7. Specific examples

There are numerous solutions for the anomalous $\mathrm{U}(1)_A$ charges, satisfying the anomaly-cancellation conditions and the symmetric mass matrix requirements we impose. In this section our intention is to show that there exist cases with natural sets of charges which lead to viable models. Since, as has been shown, there are numerous constraints that should be taken into account, we make a systematic search of solutions with large values of r, so that the μ term is sufficiently suppressed.

In tables 8–13 we present the solutions for the $\mathrm{U}(1)_A$ charges in five representative cases. The solutions for the $\mathrm{U}(1)_A$ charges in tables 8, 9 and 13 give the following

field	generation		
	1	2	3
F	3	-7	-5
$\bar{f}$	12	2	4
ℓ^c	-6	-16	-14
Higgs			
H	$-9/2$	$\bar{H}$	$-3/2$
h	10	$\bar{h}$	1

Table 12: Solution E:$k = 1, \bar{k} = 2, m = -4, n = 16$, $r = 11$.

field	generation		
	1	2	3
F	-7	-5	-3
$\bar{f}$	-8	-6	-4
ℓ^c	-6	-4	-2
Higgs			
H	$-15/2$	$\bar{H}$	$-5/2$
h	6	$\bar{h}$	7

Table 13: Solution F: $k = -9, \bar{k} = 2, m = -4, n = -8, r = 13$.

general fermion mass texture at the unification scale:

$$\frac{m_Q}{\langle h \rangle}, \frac{m_d}{\langle \bar{h} \rangle}, \frac{m_\ell}{\langle \bar{h} \rangle} \propto \begin{pmatrix} \rho^8 & \rho^6 & \rho^4 \\ \rho^6 & \rho^4 & \rho^2 \\ \rho^4 & \rho^2 & 1 \end{pmatrix}, \tag{7.1}$$

up to order one coefficients, where the parameter ρ was defined in (5.5). The case shown in table 10 predicts the following mass structure:

$$\frac{m_Q}{\langle h \rangle}, \frac{m_d}{\langle \bar{h} \rangle}, \frac{m_\ell}{\langle \bar{h} \rangle} \propto \begin{pmatrix} \rho^6 & 0 & \rho^3 \\ 0 & \rho^3 & 0 \\ \rho^3 & 0 & 1 \end{pmatrix}. \tag{7.2}$$

Finally, the solutions presented in tables 11 and 12 predict the texture

$$\frac{m_Q}{\langle h \rangle}, \frac{m_d}{\langle \bar{h} \rangle}, \frac{m_\ell}{\langle \bar{h} \rangle} \propto \begin{pmatrix} \rho^{16} & \rho^6 & \rho^8 \\ \rho^6 & \rho^4 & \rho^2 \\ \rho^4 & \rho^2 & 1 \end{pmatrix}. \tag{7.3}$$

The heavy Majorana mass matrix is given for any of the above textures, as has been shown in the previous section, by

$$\mathcal{M}_N = \rho^{|r|} M_U \frac{m_\ell}{\langle \bar{h} \rangle}, \tag{7.4}$$

where values of the parameter r are shown in the tables. All three cases have been shown to give a correct fermion mass hierarchy, so we will not elaborate this point further.[6] Instead, we discuss the B- and L-violating operators and the μ term.

To estimate the effects of the various R-violating and B-violating operators, we present also in the tables the values of the various parameters $k, \bar{k}$, etc. In the cases A, B and C, we show three examples with simple charge assignments which give

[6]For a complete list of acceptable fermion mass textures see [31]. The second texture was firstly proposed in [32]. For a numerical investigation of the first case, see [33] (for a non-string version) and [34] (for a model of string origin).

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a natural fermion mass hierarchy. However, in these three cases the R-violating terms are not sufficiently suppressed, so a further discrete symmetry is needed to avoid them. Further, the μ term is acceptable in cases A, C, but in case B it is not sufficiently suppressed, so the Higgs doublets would have unacceptably large mixing, in the absence of any further selection rule.

A solution to these problems requires more peculiar charge assignments, which are shown in the three remaining examples we present in this work. Solution D does not suppress sufficiently the μ term, but it exhibits the interesting fact that most of the B- and L-violating operators are suppressed, as shown in table 14. Moreover, the R-violating operators of the form $HFF\bar{f}$ have half-integer charges, and therefore are not present in the superpotential. In model E, the value $r = 11$ solves marginally the Higgs mixing problem. The B-violating operators $FFF\bar{f}$ are also marginally suppressed, their order of suppression being also shown in table 14. As r increases, we observe that we find solutions which satisfy both requirements, i.e. the μ term is of order m_W, whilst baryon-decay operators are also sufficiently suppressed. Solution F is such an example: the value $r = 13$ gives naturally a value for μ at the electroweak scale, whilst all the baryon-violating operators of form $FFF\bar{f}$ and $F\bar{f}\bar{f}\ell^c$ are highly suppressed, as can be seen in the last column of table 14. The rather interesting consequence of this particular charge assignment is the fact that all R-violating operators are absent, as in cases D and E.

8. Conclusions

The appearance of anomalous abelian $\mathrm{U}(1)_A$ symmetries is a generic phenomenon in string constructions. They act as family symmetries and, together with the gauge symmetry and other discrete (string) symmetries, determine the possible forms of the superpotential couplings of fermions. In this paper we have analysed in detail the implications of a general $\mathrm{U}(1)_A$ anomalous symmetry in the context of the flipped $\mathrm{SU}(5) \times \mathrm{U}(1)'$ model derivable from string.

We imposed the appropriate anomaly-cancellation conditions on the anomalous abelian charges of the $\mathrm{SU}(5) \times \mathrm{U}(1)'$ superfields, so as to obtain the canonical value for the weak mixing angle: $\sin^2\theta_W = 3/8$ at the unification scale. For simplicity, we have restricted our attention to symmetric mass textures, although non-symmetric mass matrices for the up quarks and the leptons are also a viable possibility. We further used the known bounds from low-energy experimental physics on L- and B-violating Yukawa couplings, as well as the acceptable ranges of the mixing angles and the fermion masses, to constrain the possible $\mathrm{U}(1)_A$ charges of the matter and Higgs representations of $\mathrm{SU}(5) \times \mathrm{U}(1)'$.

We have provided solutions of the anomaly-cancellation constraints and showed that we can build low-energy effective models providing the correct fermion mass spectrum with rather simple charge assignments. The success of simple, small $\mathrm{U}(1)_A$

Operator		Model D	Model E	Model F
$q_1q_1q_2\ell_1$	$d_1^cu_1^cu_2^ce_1^c$	13	11	-27
$q_1q_1q_3\ell_1$	$d_1^cu_1^cu_3^ce_1^c$	15	13	-25
$q_1q_2q_3\ell_1$	$d_1^cu_2^cu_3^ce_1^c$ $d_2^cu_1^cu_3^ce_1^c$ $d_3^cu_1^cu_2^ce_1^c$	5	3	-23
$q_2q_1q_2\ell_1$	$d_2^cu_1^cu_2^ce_1^c$	3	1	-25
$q_2q_2q_3\ell_1$	$d_2^cu_2^cu_3^ce_1^c$	-5	-7	-21
$q_3q_1q_3\ell_1$	$d_3^cu_1^cu_3^ce_1^c$	7	4	-21
$q_3q_2q_3\ell_1$	$d_3^cu_2^cu_3^ce_1^c$	-3	-5	-19
$q_1q_1q_2\ell_2$	$d_1^cu_1^cu_2^ce_2^c$	3	1	-25
$q_1q_1q_3\ell_2$	$d_1^cu_1^cu_3^ce_2^c$	5	3	-23
$q_1q_2q_3\ell_2$	$d_1^cu_2^cu_3^ce_2^c$ $d_2^cu_1^cu_3^ce_2^c$ $d_3^cu_1^cu_2^ce_2^c$	-5	-7	-21
$q_2q_1q_2\ell_2$	$d_2^cu_1^cu_2^ce_2^c$	-7	-9	-23
$q_2q_2q_3\ell_2$	$d_2^cu_2^cu_3^ce_2^c$	-15	-17	-19
$q_3q_1q_3\ell_2$	$d_3^cu_1^cu_3^ce_2^c$	-3	-5	-19
$q_3q_2q_3\ell_2$	$d_3^cu_2^cu_3^ce_2^c$	-13	-15	-17
$q_1q_1q_2\ell_3$	$d_1^cu_1^cu_2^ce_3^c$	5	3	-23
$q_1q_1q_3\ell_3$	$d_1^cu_1^cu_3^ce_3^c$	7	5	-21
$q_1q_2q_3\ell_3$	$d_1^cu_2^cu_3^ce_3^c$ $d_2^cu_1^cu_3^ce_3^c$ $d_3^cu_1^cu_2^ce_3^c$	-3	-5	-19
$q_2q_1q_2\ell_3$	$d_2^cu_1^cu_2^ce_3^c$	-5	-7	-21
$q_2q_2q_3\ell_3$	$d_2^cu_2^cu_3^ce_3^c$	-13	-15	-17
$q_3q_1q_3\ell_3$	$d_3^cu_1^cu_3^ce_3^c$	$-1!$	-3	-17
$q_3q_2q_3\ell_3$	$d_3^cu_2^cu_3^ce_3^c$	-11	-13	-15

Table 14: Dimension-5 proton decay operators originating from the $\mathrm{SU}(5)\times\mathrm{U}(1)'$ invariants $FFF\bar{f}$ and $F\bar{f}\bar{f}\ell^c$ are presented in the first and second columns. In columns 3–5, we present the numerical values of the $\mathrm{U}(1)_A$ charges (and therefore the powers of the corresponding non-renormalizable terms) for models D, E and F respectively.

charges is spoiled, however, by the appearance of R-violating interactions, as well as B-violating decays at unacceptable levels and a large mixing term μ for the ordinary Higgs doublets. Nevertheless, we find models which are compatible with all the known experimental facts, albeit with relatively large values for the $\mathrm{U}(1)_A$ charges. The R-violating terms are prohibited, and baryon- and lepton-violating operators are highly suppressed.

In the present work, we have shown how the anomalous $\mathrm{U}(1)_A$ in string-derivable models based on intermediate gauge symmetries may contribute in the construction of a phenomenologically successful theory. Although we have in mind string-derived models, in order to make our analysis as general as possible, we have incorporated only the most common features of present-day string constructions. We have relaxed several of the simplified assumptions often used in the recent literature, but, at this stage of our approach, we have retained some which may not give a fully realistic picture. In the rest of this section, we discuss briefly these points and the limitations of our procedure, and make a few comments on possible future extensions of our analysis.

A crucial point for the consistency of the above symmetric scheme is that the expansion parameter λ in the up-quark sector differs from the corresponding parameter ϵ in the down-quark sector. In the context of the MSSM, $\lambda = \phi/M_1$ and $\varepsilon = \phi/M_2$, where M_1 and M_2 are related to the scales where the up- and down-quark mass matrices are formed. The viability of most phenomenological explorations of $\mathrm{U}(1)$ family-symmetry models is based on the observation [14] that it is possible for additional vector-like Higgs pairs to acquire their mass via spontaneous breaking after compactification. Then, a strong violation of the $\mathrm{SU}(2)_R$ symmetry of the quark sector may occur, and as a result $M_1 \neq M_2$ [14]. This provides the possibility that $\lambda \neq \varepsilon$, and a correct mass hierarchy may result.

In usual compactification scenarios, however, including also the cases of the free-fermionic string models, this is not the case. In fact, there is only one scale: $M_1 = M_2 \equiv M_{\text{string}}$, and therefore the above two parameters coincide: $\lambda = \epsilon$ and consequently $\bar{\lambda} = \bar{\epsilon}$. Hence, if we are not able to distinguish the up- and down-quark expansion mass parameters, it is impossible to obtain a correct hierarchical mass pattern.

In realistic string scenarios, the discrimination between the various fermion mass matrices is based on completely different observations. There are basically three sources of the breaking of the up-/down-quark mass matrix symmetry. First, there are many singlet fields acquiring vevs which couple differently to the various types of quarks. Secondly, there are multiple $\mathrm{U}(1)$ symmetries, although only one is anomalous. Finally, fermion fields are not always charged "symmetrically", eventually leading to non-symmetric mass matrices whenever the non-abelian structure of the model makes this possible. In the present case of the $\mathrm{SU}(5) \times \mathrm{U}(1)$ model, only the down-quark mass matrix is always symmetric, the reason being that the left-handed $\mathrm{SU}(2)$-doublet quarks and the right-handed $\mathrm{SU}(2)$-singlet down quarks belong to the same $\mathrm{SU}(5) \times \mathrm{U}(1)$ representations. On the other hand, the up-quark and charged-lepton mass matrices need not be symmetric.

A question one should then answer is: what are the minimal modifications of the above scenario leading to a natural hierarchical mass spectrum? If our aim is to obtain an economical way of constructing viable mass matrices, we should avoid introducing a large number of singlets or symmetries. Thus, in more realistic cases, in order to emulate a realistic string scenario, one should relax the symmetry constraints on the U(1) charges of the up-quark mass matrix. On the other hand, one may retain the same number of singlets and employ a single anomalous U(1) family symmetry, if this is sufficient to obtain a satisfactory result. We plan to analyse this scenario in a future publication.

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